RESEARCH ARTICLE OPEN ACCESS

On Some Double Integrals of $\overline{H}$-Function of Two Variables and Their Applications

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Abstract
This paper deals with the evaluation of four integrals of $\overline{H}$-function of two variables proposed by Singh and Mandia [7] and their applications in deriving double half-range Fourier series for the $\overline{H}$-function of two variables. A multiple integral and a multiple half-range Fourier series of the $\overline{H}$-function of two variables are derived analogous to the double integral and double half-range Fourier series of the $\overline{H}$-function of two variables.

Key words: $\overline{H}$-function of two variables, Half-range Fourier series, $\overline{H}$-function, Multiple half-range Fourier series.
(2000 Mathematics Subject Classification: 33C99)

I. Introduction

The $\overline{H}$-function of two variables will be defined and represented by Singh and Mandia [7] in the following manner:

$$\overline{H}[x,y]=\overline{H}\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]=\overline{H}^{o,n_1:\ m_2,n_2:m_3,n_2}_{p_1,q_1:p_2,q_2;p_2,q_2}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;K_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2},(e_j,E_j;R_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3}\\(b_j,\beta_j;B_j)_{1,q_1},(d_j,\delta_j)_{1,m_2},(d_j,\delta_j;L_j)_{m_2+1,q_2},(f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3}\end{matrix}\right.\right]$$

$$=-\frac{1}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\phi_2(\xi)\phi_3(\eta)x^{\xi}y^{\eta}d\xi d\eta \tag{1.1}$$

Where

$$\phi_1(\xi,\eta)=\frac{\prod_{j=1}^{n_1}\Gamma\left(1-a_j+\alpha_j\xi+A_j\eta\right)}{\prod_{j=n_1+1}^{p_1}\Gamma\left(a_j-\alpha_j\xi-A_j\eta\right)\prod_{j=1}^{q_1}\Gamma\left(1-b_j+\beta_j\xi+B_j\eta\right)} \tag{1.2}$$

$$\phi_2(\xi)=\frac{\prod_{j=1}^{n_2}\left\{\Gamma\left(1-c_j+\gamma_j\xi\right)\right\}^{K_j}\prod_{j=1}^{m_2}\Gamma\left(d_j-\delta_j\xi\right)}{\prod_{j=n_2+1}^{p_2}\Gamma\left(c_j-\gamma_j\xi\right)\prod_{j=m_2+1}^{q_2}\left\{\Gamma\left(1-d_j+\delta_j\xi\right)\right\}^{L_j}} \tag{1.3}$$

$$\phi_3(\eta)=\frac{\prod_{j=1}^{n_3}\left\{\Gamma\left(1-e_j+E_j\eta\right)\right\}^{R_j}\prod_{j=1}^{m_3}\Gamma\left(f_j-F_j\eta\right)}{\prod_{j=n_3+1}^{p_3}\Gamma\left(e_j-E_j\eta\right)\prod_{j=m_3+1}^{q_3}\left\{\Gamma\left(1-f_j+F_j\eta\right)\right\}^{S_j}} \tag{1.4}$$

Where x and y are not equal to zero (real or complex), and an empty product is interpreted as unity p_i,q_i,n_i,m_j are non-negative integers such that $0\le n_i\le p_i, o\le m_j\le q_j (i=1,2,3; j=2,3)$. All the $a_j(j=1,2,...,p_1), b_j(j=1,2,...,q_1), c_j(j=1,2,...,p_2), d_j(j=1,2,...,q_2),$

$e_j(j=1,2,...,p_3),\ f_j(j=1,2,...,q_3)$ are complex parameters.
$\gamma_j \geq 0(j=1,2,...,p_2), \delta_j \geq 0(j=1,2,...,q_2)$ (not all zero simultaneously), similarly $E_j \geq 0(j=1,2,...,p_3), F_j \geq 0(j=1,2,...,q_3)$ (not all zero simultaneously). The exponents $K_j(j=1,2,...,n_3), L_j(j=m_2+1,...,q_2), R_j(j=1,2,...,n_3), S_j(j=m_3+1,...,q_3)$ can take on non-negative values.

The contour L_1 is in ξ-plane and runs from $-i\infty$ to $+i\infty$. The poles of $\Gamma\left(d_j-\delta_j\xi\right)(j=1,2,...,m_2)$ lie to the right and the poles of $\Gamma\left\{\left(1-c_j+\gamma_j\xi\right)\right\}^{K_j}(j=1,2,...,n_2), \Gamma\left(1-a_j+\alpha_j\xi+A_j\eta\right)(j=1,2,...,n_1)$ to the left of the contour. For $K_j(j=1,2,...,n_2)$ not an integer, the poles of gamma functions of the numerator in (1.3) are converted to the branch points.

The contour L_2 is in η-plane and runs from $-i\infty$ to $+i\infty$. The poles of $\Gamma\left(f_j-F_j\eta\right)(j=1,2,...,m_3)$ lie to the right and the poles of
$\Gamma\left\{\left(1-e_j+E_j\eta\right)\right\}^{R_j}(j=1,2,...,n_3), \Gamma\left(1-a_j+\alpha_j\xi+A_j\eta\right)(j=1,2,...,n_1)$ to the left of the contour. For $R_j(j=1,2,...,n_3)$ not an integer, the poles of gamma functions of the numerator in (1.4) are converted to the branch points.
The functions defined in (1.1) is an analytic function of x and y, if

$$U=\sum_{j=1}^{p_1}\alpha_j+\sum_{j=1}^{p_2}\gamma_j-\sum_{j=1}^{q_1}\beta_j-\sum_{j=1}^{q_2}\delta_j<0 \tag{1.5}$$

$$V=\sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_3}E_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_3}F_j<0 \tag{1.6}$$

The integral in (1.1) converges under the following set of conditions:

$$\Omega=\sum_{j=1}^{n_1}\alpha_j-\sum_{j=n_1+1}^{p_1}\alpha_j+\sum_{j=1}^{m_2}\delta_j-\sum_{j=m_2+1}^{q_2}\delta_jL_j+\sum_{j=1}^{n_2}\gamma_jK_j-\sum_{j=n_2+1}^{p_2}\gamma_j-\sum_{j=1}^{q_1}\beta_j>0 \tag{1.7}$$

$$\Lambda=\sum_{j=1}^{n_1}A_j-\sum_{j=n_1+1}^{p_1}A_j+\sum_{j=1}^{m_2}F_j-\sum_{j=m_2+1}^{q_2}F_jS_j+\sum_{j=1}^{n_3}E_jR_j-\sum_{j=n_2+1}^{p_3}E_j-\sum_{j=1}^{q_1}B_j>0 \tag{1.8}$$

$$|\arg x|<\frac{1}{2}\Omega\pi,\ |\arg y|<\frac{1}{2}\Lambda\pi \tag{1.9}$$

The behavior of the $\overline{H}$-function of two variables for small values of $|z|$ follows as:

$$\overline{H}[x,y]=0(|x|^{\alpha}|y|^{\beta}), \max\{|x|,|y|\}\to 0 \tag{1.10}$$

Where

$$\alpha=\min_{1\leq j\leq m_2}\left[\operatorname{Re}\left(\frac{d_j}{\delta_j}\right)\right] \qquad \beta=\min_{1\leq j\leq m_2}\left[\operatorname{Re}\left(\frac{f_j}{F_j}\right)\right] \tag{1.11}$$

For large value of $|z|$,

$$\overline{H}[x,y]=0\left\{|x|^{\alpha'},|y|^{\beta'}\right\}, \min\{|x|,|y|\}\to 0 \tag{1.12}$$

Where

$$\alpha'=\max_{1\leq j\leq n_2}\operatorname{Re}\left(K_j\frac{c_j-1}{\gamma_j}\right),\ \beta'=\max_{1\leq j\leq n_3}\operatorname{Re}\left(R_j\frac{e_j-1}{E_j}\right) \tag{1.13}$$

Provided that $U<0$ and $V<0$.

If we take
$K_j = 1(j = 1, 2, ..., n_2), L_j = 1(j = m_2 + 1, ..., q_2), R_j = 1(j = 1, 2, ..., n_3), S_j = 1(j = m_3 + 1, ..., q_3)$ in (1.1), the $\overline{H}$ -function of two variables reduces to H -function of two variables due to [4].
Orthogonality of Sine and Cosine functions:

$$\int_0^{\pi} \sin mx \sin nx dx = \begin{cases} 0; & m \neq n \\ \frac{\pi}{2}; & m = n \end{cases} \tag{1.14}$$

$$\int_0^{\pi} \cos mx \cos nx dx = \begin{cases} 0; & m \neq n \\ \frac{\pi}{2}; & m = n \\ \pi; & m = n = 0 \end{cases} \tag{1.15}$$

II. Double Integrals

The following double integrals has been evaluated in this paper

$$\int_0^{\pi}\int_0^{\pi} (\sin x)^{\lambda-1} (\sin y)^{\mu-1} \sin rx \sin ty \, h(x, y) \, dxdy$$
$$= 2^{2-\lambda-\mu} \pi^2 \sin\left(\frac{r\pi}{2}\right) \sin\left(\frac{t\pi}{2}\right) \psi(r, t) \tag{2.1}$$

$$\int_0^{\pi}\int_0^{\pi} (\sin x)^{\lambda-1} (\sin y)^{\mu-1} \sin rx \cos ty \, h(x, y) \, dxdy$$
$$= 2^{2-\lambda-\mu} \pi^2 \sin\left(\frac{r\pi}{2}\right) \cos\left(\frac{t\pi}{2}\right) \psi(r, t) \tag{2.2}$$

$$\int_0^{\pi}\int_0^{\pi} (\sin x)^{\lambda-1} (\sin y)^{\mu-1} \cos rx \sin ty \, h(x, y) \, dxdy$$
$$= 2^{2-\lambda-\mu} \pi^2 \cos\left(\frac{r\pi}{2}\right) \sin\left(\frac{t\pi}{2}\right) \psi(r, t) \tag{2.3}$$

$$\int_0^{\pi}\int_0^{\pi} (\sin x)^{\lambda-1} (\sin y)^{\mu-1} \cos rx \cos ty \, h(x, y) \, dxdy$$
$$= 2^{2-\lambda-\mu} \pi^2 \cos\left(\frac{r\pi}{2}\right) \cos\left(\frac{t\pi}{2}\right) \psi(r, t) \tag{2.4}$$

Where

$$h(x, y) = \overline{H}^{o,n_1:\, m_2,n_2:m_3,n_3}_{p_1,q_1:p_2,q_2;p_2,q_2} \left[\begin{matrix} u(\sin x)^{2c} \\ v(\sin y)^{2d} \end{matrix} \middle| \begin{matrix} (a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;K_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2},(e_j,E_j;R_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3} \\ (b_j,\beta_j;B_j)_{1,q_1},(d_j,\delta_j)_{1,m_2},(d_j,\delta_j;L_j)_{m_2+1,q_2},(f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3} \end{matrix} \right]$$

And

$$\psi(r, t) =$$

$$\overline{H}^{o,n_1+2:\, m_2,n_2:m_3,n_2}_{p_1+2,q_1+4:p_2,q_2;p_2,q_2} \left[\begin{matrix} u2^{2(c+d)} \\ v \end{matrix} \middle| \begin{matrix} (1-\lambda,2c;1),(1-\mu,2d;1),(a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;K_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2},(e_j,E_j;R_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3} \\ (b_j,\beta_j;B_j)_{1,q_1},\left(\frac{1-\lambda\pm r}{2},c;1\right),\left(\frac{1-\mu\pm t}{2},d;1\right),(d_j,\delta_j)_{1,m_2},(d_j,\delta_j;L_j)_{m_2+1,q_2},(f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3} \end{matrix} \right]$$

And $(\lambda \pm \mu)$ stands for the pair of parameters $(\lambda + \mu), (\lambda - \mu)$.

Also $\text{Re}(\lambda) + 2c \min\limits_{1 \le j \le m_1} \left(\frac{b_j}{\beta_j}\right) + 2c \min\limits_{1 \le j \le m_2} \left(\frac{d_j}{\delta_j}\right) > 0$,

$\text{Re}(\mu)+2d\min_{1\le j\le m_1}\left(\frac{b_j}{\beta_j}\right)+2c\min_{1\le j\le m_2}\left(\frac{d_j}{\delta_j}\right)>0$ and conditions (1.7), (1.8) and (1.9) are also satisfied.

Proof: If we express the $\overline{H}$ -function of two variables occurring in the integrand of (2.1) as the Mellin-Barnes type integral (1.1) and interchange the order of integrations (which is permissible due to the absolute convergence of the integrals involved in the process), we get
L.H.S. of (2.1)

$$=-\frac{1}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\phi_2(\xi)\phi_3(\eta)u^{\xi}v^{\eta}$$

$$\left[\int_0^{\pi}(\sin x)^{\mu+2c\xi-1}\sin rx dx\int_0^{\pi}(\sin y)^{\mu+2d\xi-1}\sin ty dy\right]d\xi d\eta$$

Now, applying the result ([5],p.70, eq. (3.1.5)) and equation (1.1), the result (2.1) follows at once. The remaining integrals can be evaluated similarly.

III. Double Half-Range Fourier Series

The following double half-range Fourier series will be proved:

$$f(x,y)=2^{4-\lambda-\mu}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{n\pi}{2}\right)\psi(m,n)\sin mx\sin ny \tag{3.1}$$

$$f(x,y)=2^{4-\lambda-\mu}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\sin\left(\frac{m\pi}{2}\right)\cos\left(\frac{n\pi}{2}\right)\psi(m,n)\sin mx\cos ny \tag{3.2}$$

$$f(x,y)=2^{4-\lambda-\mu}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\cos\left(\frac{m\pi}{2}\right)\sin\left(\frac{n\pi}{2}\right)\psi(m,n)\cos mx\sin ny \tag{3.3}$$

$$f(x,y)=2^{4-\lambda-\mu}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\cos\left(\frac{m\pi}{2}\right)\cos\left(\frac{n\pi}{2}\right)\psi(m,n)\cos mx\cos ny \tag{3.4}$$

Where $f(x,y)=(\sin x)^{\lambda-1}(\sin y)^{\mu-1}h(x,y)$ and provided that

$\text{Re}(\lambda)+2c\min_{1\le j\le m_1}\left(\frac{b_j}{\beta_j}\right)+2c\min_{1\le j\le m_2}\left(\frac{d_j}{\delta_j}\right)>0$, $\text{Re}(\mu)+2d\min_{1\le j\le m_1}\left(\frac{b_j}{\beta_j}\right)+2c\min_{1\le j\le m_2}\left(\frac{d_j}{\delta_j}\right)>0$ and conditions (1.7), (1.8) and (1.9) are also satisfied.

Proof: To prove (3.1), let

$$f(x,y)=(\sin x)^{\lambda-1}(\sin y)^{\mu-1}h(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{m,n}\sin mx\sin ny \tag{3.5}$$

Which is valid since $f(x,y)$ is continuous and of bounded variation in the open interval $(0,\pi)$.

Multiplying both sides of (3.5) by $\sin rx\sin ty$ and integrating from 0 to π with respect to both x and y, it is seen that

$$\int_0^{\pi}\int_0^{\pi}(\sin x)^{\lambda-1}(\sin y)^{\mu-1}h(x,y)\sin rx\sin ty dx dy=$$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{m,n}\int_0^{\pi}\int_0^{\pi}\sin mx\sin ny\sin rx\sin ty dx dy \tag{3.6}$$

Now using (2.1), and orthogonal property of sine functions, it follows that

$$A_{r,t}=2^{4-\lambda-\mu}\sin\left(\frac{r\pi}{2}\right)\sin\left(\frac{t\pi}{2}\right)\psi(r,t) \tag{3.7}$$

Substituting the value of $A_{m,n}$ from (3.7) to (3.5), the result (3.1) follows at once.

To establish (3.2), put

$$f(x,y)=\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}B_{m,n}\sin mx\cos ny \tag{3.8}$$

Multiplying both sides of (3.8) by $\sin rx\cos ty$ and integrating from 0 to π with respect to both x and y and using (2.2) and orthogonal properties of sine and cosine functions, we find that

$$B_{r,t}=2^{4-\lambda-\mu}\sin\left(\frac{r\pi}{2}\right)\cos\left(\frac{t\pi}{2}\right)\psi(r,t) \tag{3.9}$$

Except that $B_{r,0}$ is one-half of the above value.

From (3.8) and (3.9), the series (3.2) follows easily. The result (3.3) can be established in a manner similar to above.

To establish (3.4), let

$$f(x,y)=\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}D_{m,n}\cos mx\cos ny \tag{3.10}$$

Multiplying both sides of (3.8) by $\cos rx\cos ty$ and integrating from 0 to π with respect to both x and y and using (2.2) and orthogonal properties of cosine functions, we obtain

$$D_{r,t}=2^{4-\lambda-\mu}\cos\left(\frac{r\pi}{2}\right)\cos\left(\frac{t\pi}{2}\right)\psi(r,t) \tag{3.11}$$

Except that $D_{0,t}, D_{r,0}$ are one-half and $D_{0,0}$ is quarter of the above values.

The Fourier series (3.4) now follows from (3.10) and (3.11).

IV. Multiple Integrals

The following multiple integral analogous to (2.1) can be derived easily on following the procedure as given $2 and taking the help of ([5],p.70,eq.(3.1.5):

$$\int_0^{\pi}...\int_0^{\pi}(\sin x_1)^{\lambda_1-1}(\sin x_n)^{\lambda_n-1}\sin r_1x_1...\sin r_nx_n\, h(x_1,...,x_n)\,dx_1...dx_n$$

$$=2^{n-\lambda_1-...-\lambda_n}\pi^n\sin\left(\frac{r_1\pi}{2}\right)...\sin\left(\frac{r_n\pi}{2}\right)\psi(r_1,...,r_n) \tag{4.1}$$

Where $\mathrm{Re}(\lambda_i)+2c\min_{1\le j\le m_1}\left(\frac{b_j}{\beta_j}\right)+2c\min_{1\le j\le m_2}\left(\frac{d_j}{\delta_j}\right)>0$; $i=1,2,...,n$ and conditions (1.7), (1.8) and (1.9) are also hold, and

$$h(x_1,...,x_n)=\overline{H}^{o,n_1:\ m_2,n_2:m_3,n_2}_{p_1,q_1:p_2,q_2;p_2,q_2}\left[\begin{matrix}uv(\sin x_1)^{2c_1}\\ \vdots\\ (\sin x_n)^{2c_n}\end{matrix}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;K_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2},(e_j,E_j;R_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3}\\ (b_j,\beta_j;B_j)_{1,q_1},(d_j,\delta_j)_{1,m_2},(d_j,\delta_j;L_j)_{m_2+1,q_2},(f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3}\end{matrix}\right.\right]$$

And

$$\psi(r_1,...,r_n)=$$

$$\overline{H}^{o,n_1+2:\ m_2,n_2:m_3,n_2}_{p_1+2,q_1+4:p_2,q_2;p_2,q_2}\left[\begin{matrix}u\,2^{2(c_1+...+c_n)}\\ v\end{matrix}\left|\begin{matrix}(1-\lambda_j,2c_j;1)_{1,n},(a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;K_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2},(e_j,E_j;R_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3}\\ (b_j,\beta_j;B_j)_{1,q_1},\left(\frac{1-\lambda_j\pm r_j}{2},c_j;1\right)_{1,n},(d_j,\delta_j)_{1,m_2},(d_j,\delta_j;L_j)_{m_2+1,q_2},(f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3}\end{matrix}\right.\right]$$

For $j=1,2,...,n$.

Multiple integrals analogous to (2.2) to (2.4) can also be written easily.

V. Multiple Half-Range Fourier Series

The following multiple half-range Fourier series analogous to (2.1) can be derived on following lines as given in $3 [5], using the integral (4.1) and the multiple orthogonal property of sine functions:

$$f(x_1,...,x_n) = 2^{2n-\lambda_1-...-\lambda_n} \sum_{m_1=1}^{\infty} ... \sum_{m_n=1}^{\infty} \sin\left(\frac{m_1\pi}{2}\right) ... \sin\left(\frac{m_n\pi}{2}\right)$$

$$\psi(m_1,...,m_n) \sin(m_1x_1)...\sin(m_nx_n)$$

Where $\mathrm{Re}(\lambda_i) + 2c \min_{1\le j\le m_1}\left(\frac{b_j}{\beta_j}\right) + 2c \min_{1\le j\le m_2}\left(\frac{d_j}{\delta_j}\right) > 0 \,; i = 1, 2, ..., n$ and conditions (1.7), (1.8) and (1.9) are also satisfied , and $f(x_1,...,x_n) = (\sin x_1)^{\lambda_1-1}...(\sin x_n)^{\lambda_n-1} h(x_1,...,x_n)$ Similarly the multiple half-range Fourier series analogous to (2.2), (2.3) and (2.4) can also be solved.

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